#11 : Soft Rescue

Under certain circumstances, soap bubbles don’t break when they fall on a soft carpet.
- Investigate this **phenomenon**
- What is the **maximal landing speed** that a bubble can survive for a given carpet?

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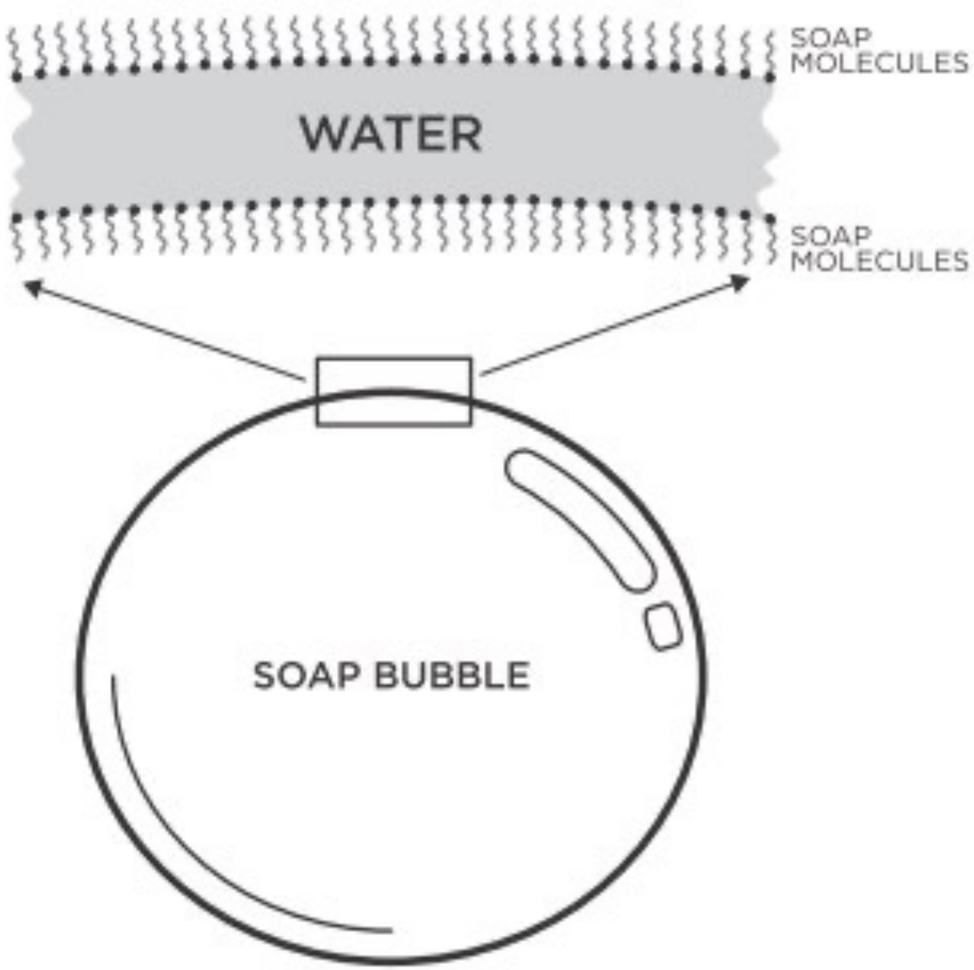
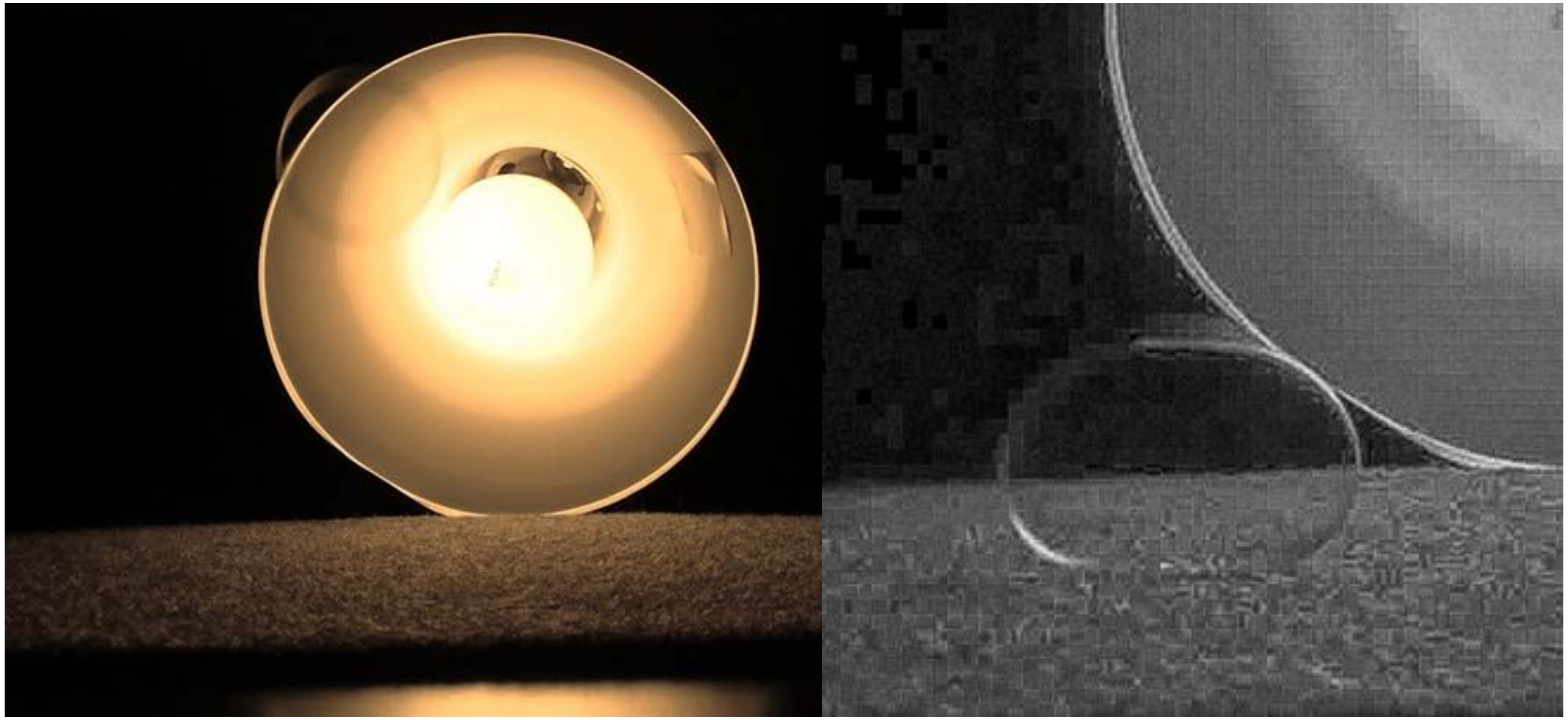
What Makes a Soap Bubble



- A thin layer of water sandwiched between two layers of soap molecules
- This structure create surface tension which allows the bubble to hold its shape

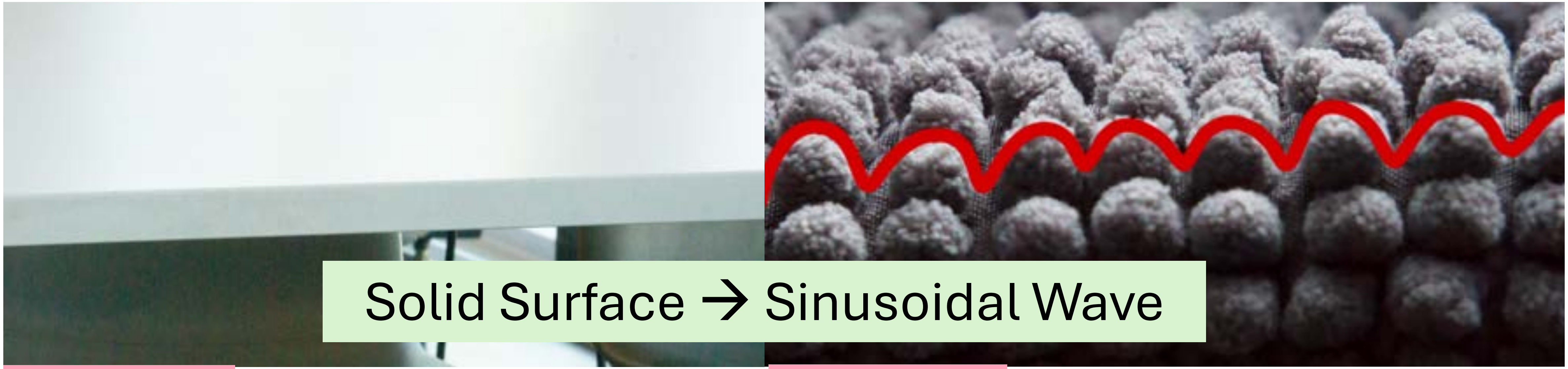
Photo from © ASTC Science World Society 2025

Low Velocity Regime



- At low velocity there is a minimal vertical compression maintaining the equilibrium of the bubble and prevent rupture

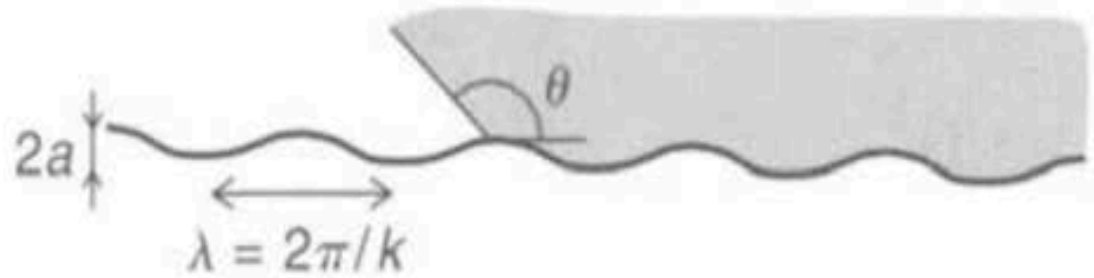
Modeling the Soft Carpet Surface

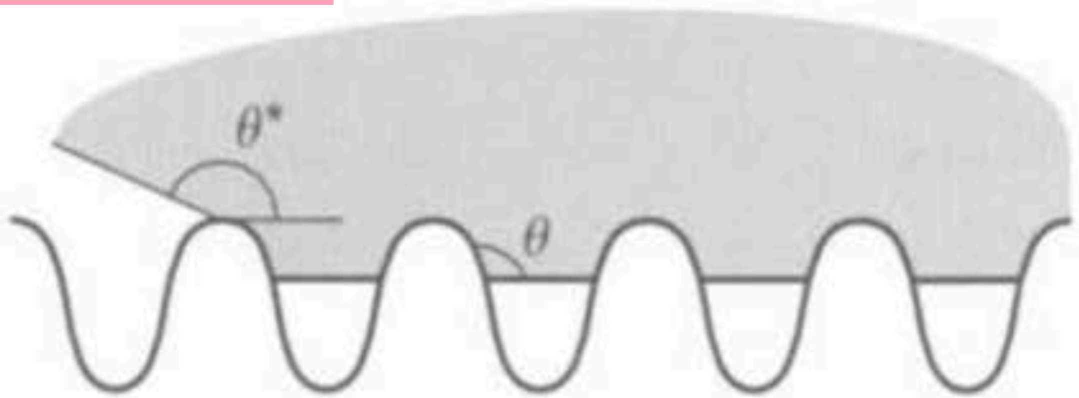


Solid Surface → Sinusoidal Wave

Low Amplitude

High Amplitude





Gennes et al, page 223

de Gennes, P.-G., Brochard-Wyart, F., & Quéré, D. (2004). *Capillarity and Wetting Phenomena*. Springer.

Why does contact angle matter?

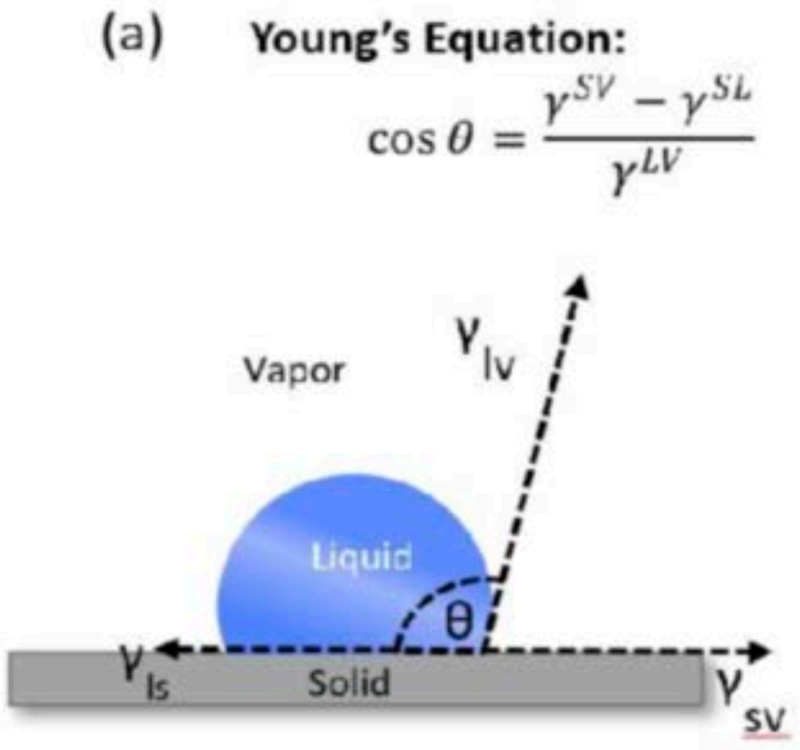
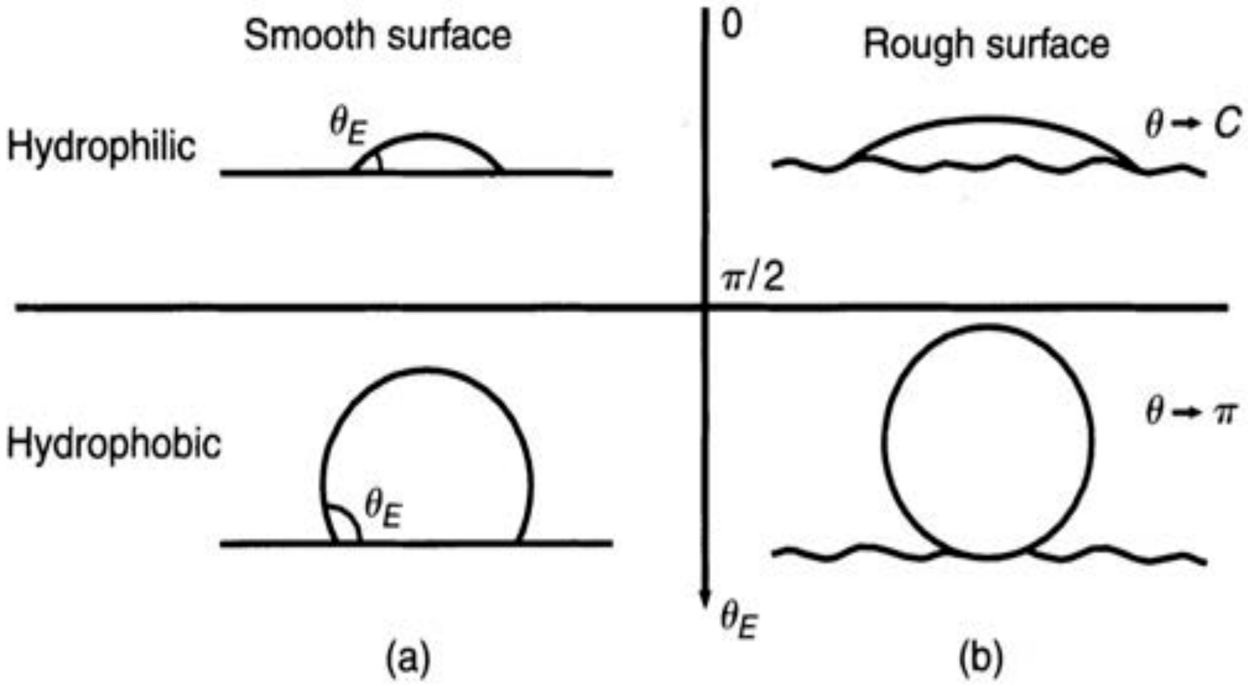


Figure 1 from Iqbal (2023), DOI: 10.20944



Gennes et al, page 24

Iqbal, Saira - *Fabrication and Characterization of Durable Superhydrophobic and Superoleophobic Surfaces on Stainless Steel Mesh Substrates.* 10.20944 (2023)

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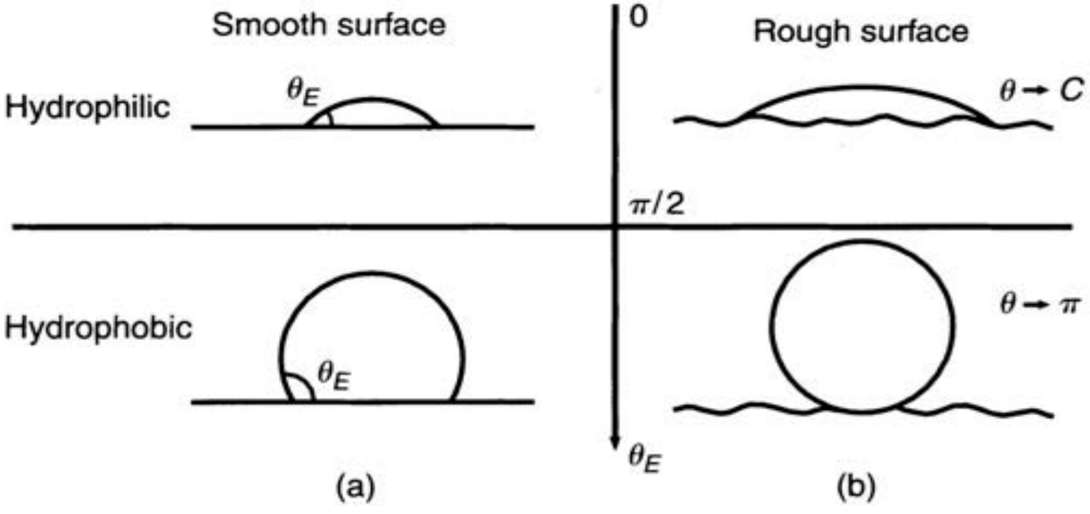
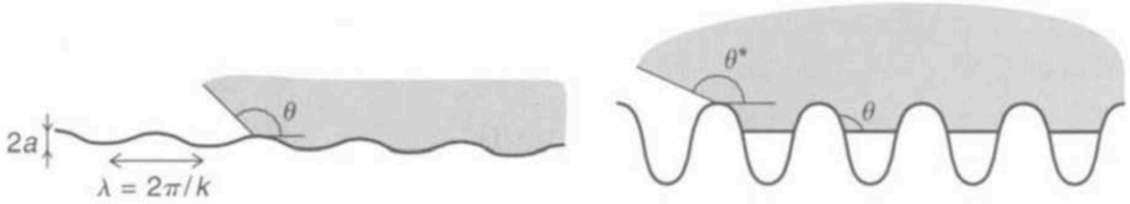


Gennes et al, page 24



Gennes et al, page 223

W_{SL} - **work of adhesion** is defined as the energy required to separate the solid liquid interface per unit area

Young-Dupré Equation

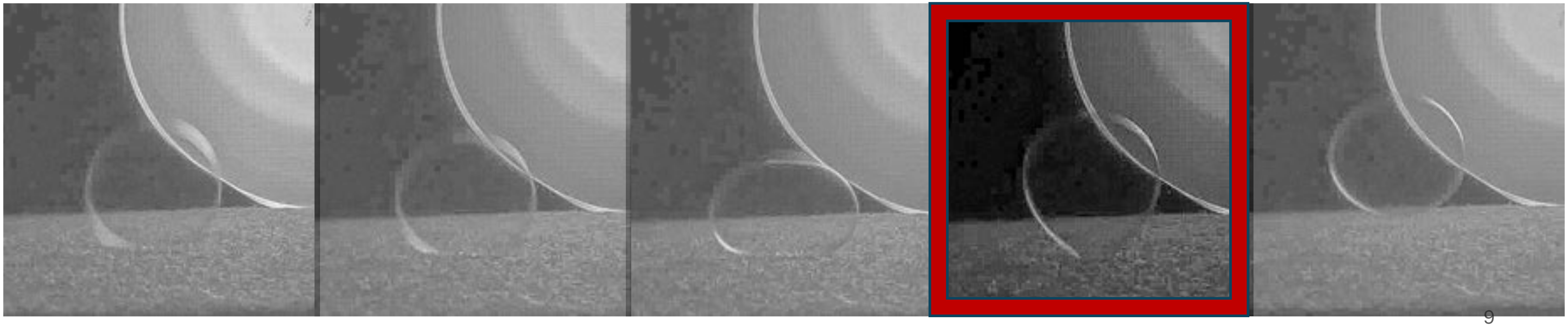
$$W_{SL} = \gamma_L(1 + \cos\theta_Y)$$

Work of Adhesion

- Higher contact angle (θ_Y) lower work of adhesion
- Smaller contact area smaller adhesion forces

Young-Dupré Equation

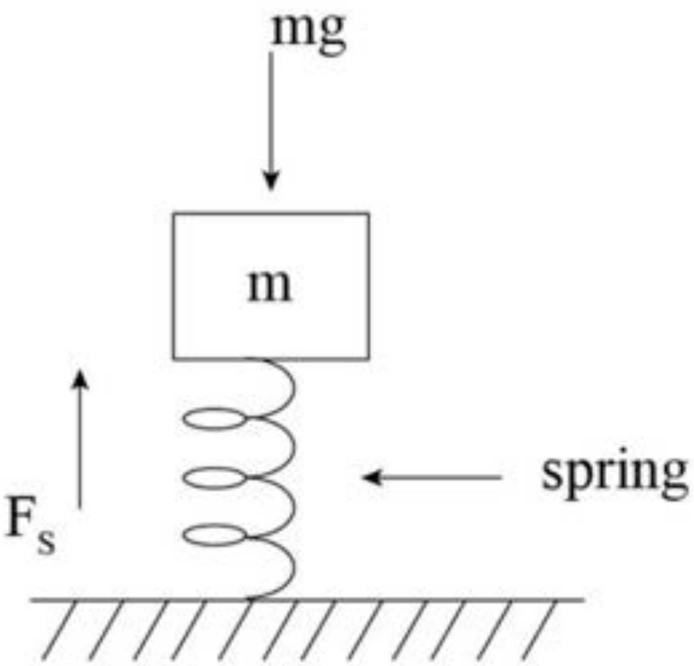
$$W_{SL} = \gamma_L(1 + \cos\theta_Y)$$



Modeling the Carpet as a Spring System

- Each carpet fiber acts like a spring
- The entire carpet is a system of n springs per unit area
- The carpet absorb impact energy, reducing stress on the bubble
- Apply Hooke's Law:

$$F_{\text{elastic}} = -kx$$



Conclusion

- Sandwich structure of the soap solution permits **surface expansion** of the soap bubble
- At **low velocity** and low vertical compression the bubble is able to maintain its equilibrium shape
- Carpet surface has **high amplitude** for the sinusoidal wave:

  - Increase contact angle

  - Decrease contact area
- Reduce the **adhesion force**
- **Spring like system** reduce stress on the bubble

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How does bubble behave when landing?

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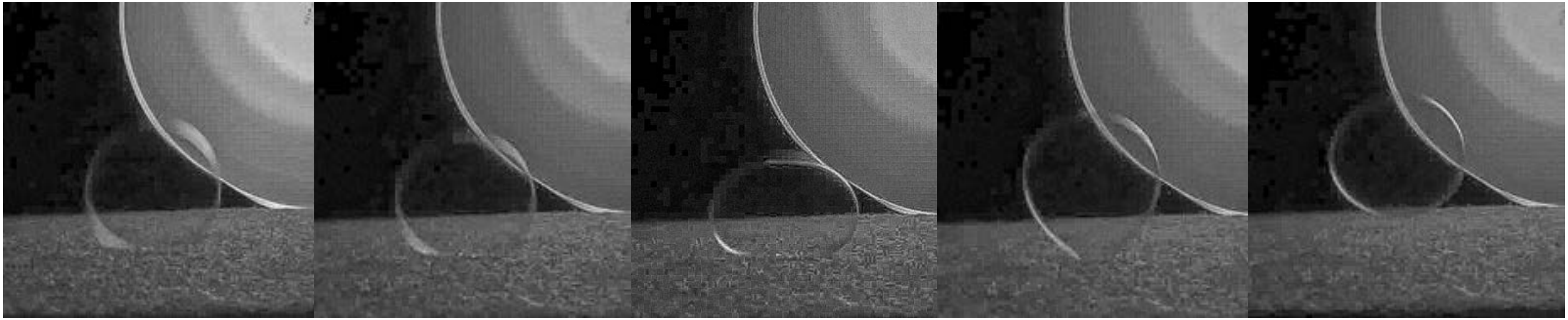

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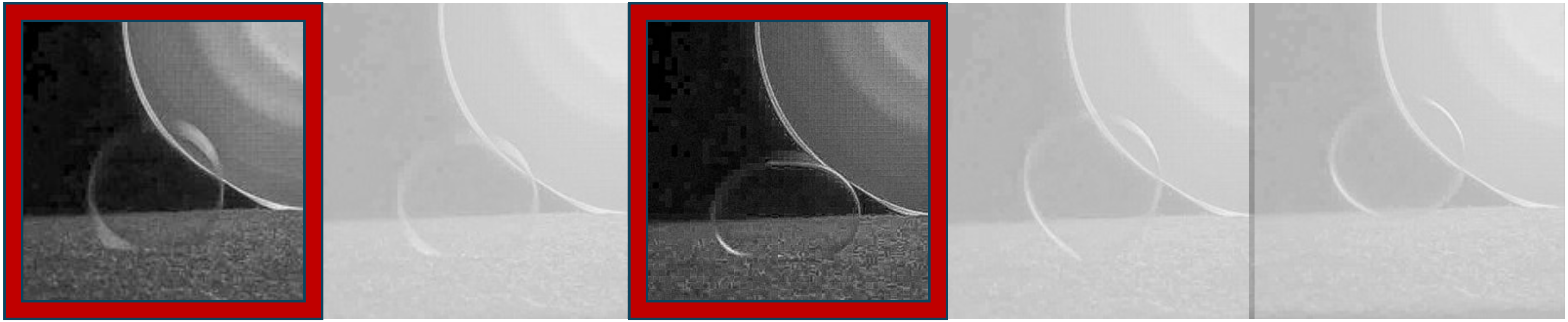

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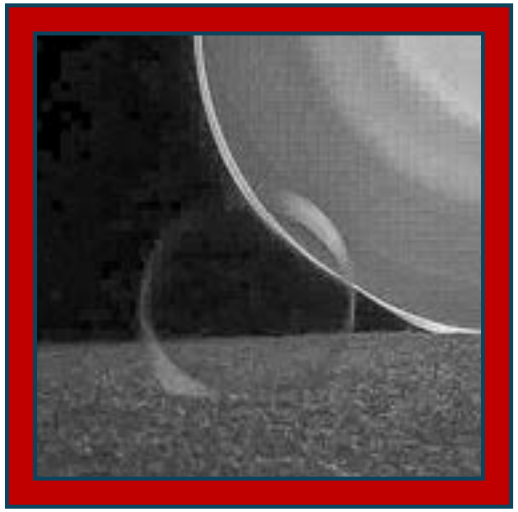
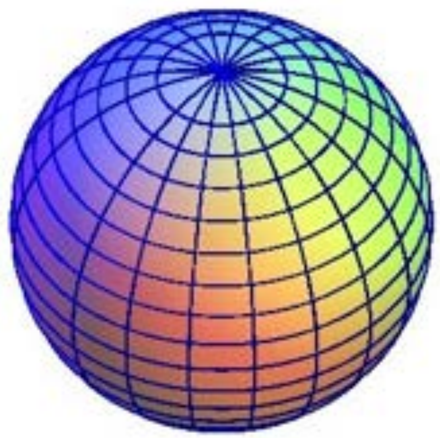
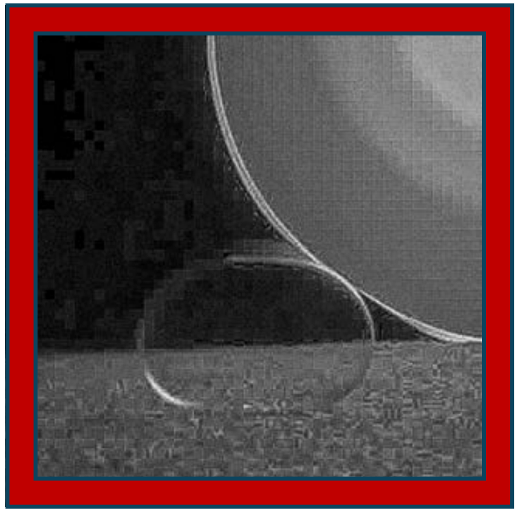
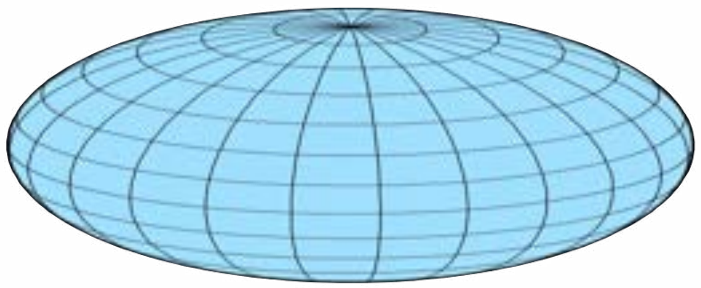




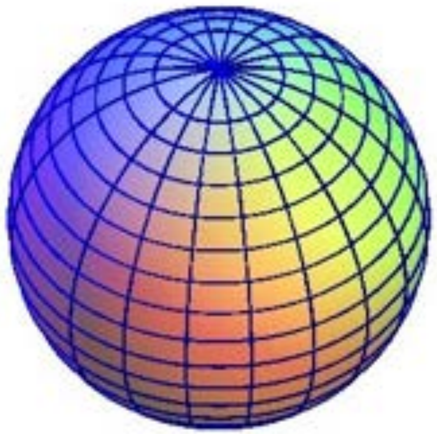
Sphere





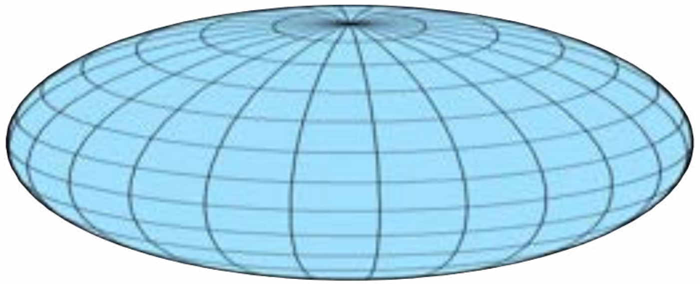
Oblate Spheroid

How does bubble behave when landing?



$$V_0 = V_f$$

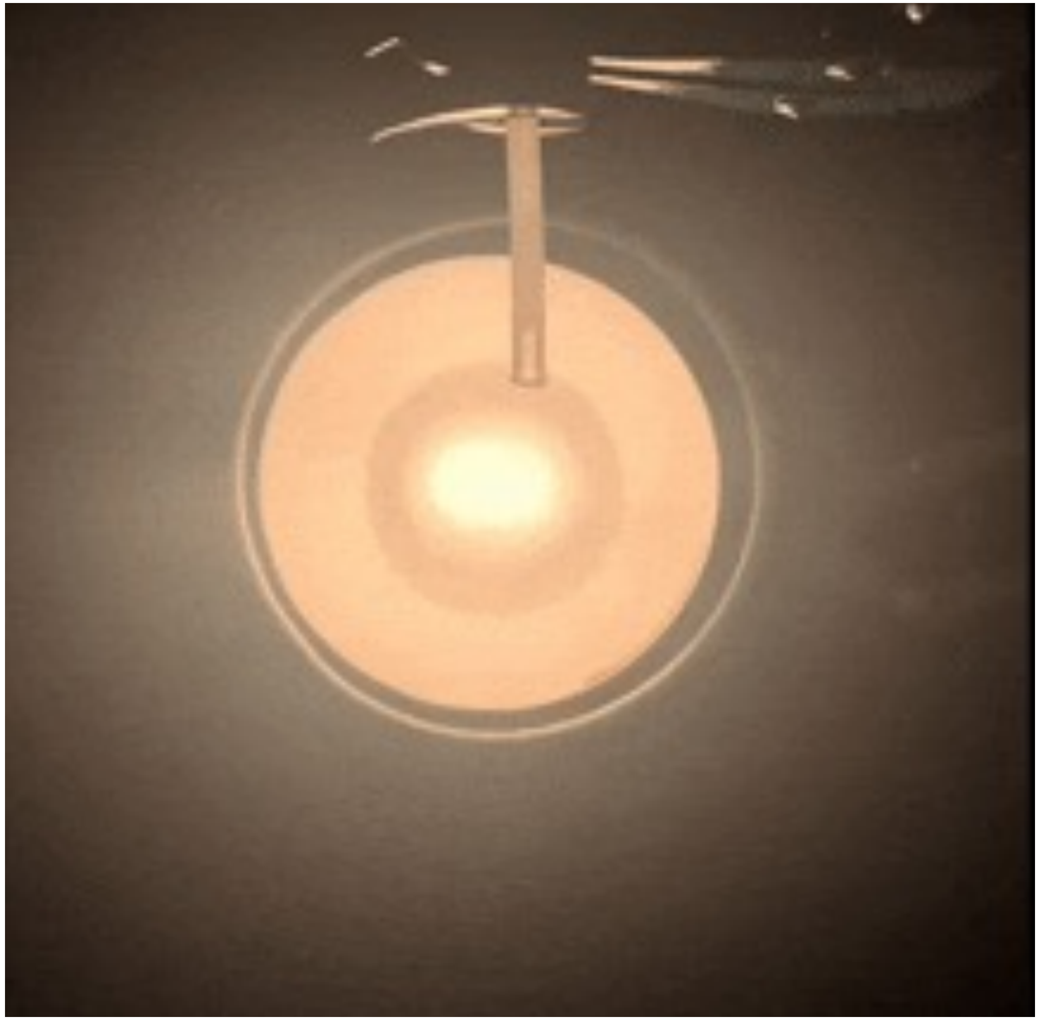
$$S_0 < S_f$$



Sphere

Oblate Spheroid

Why does the bubble break?



Increase Surface Area

↓

Decrease Film Thickness

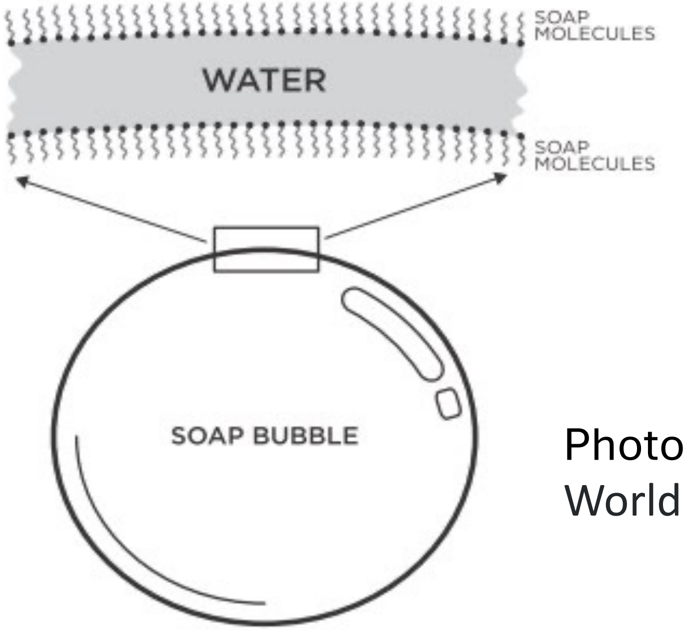
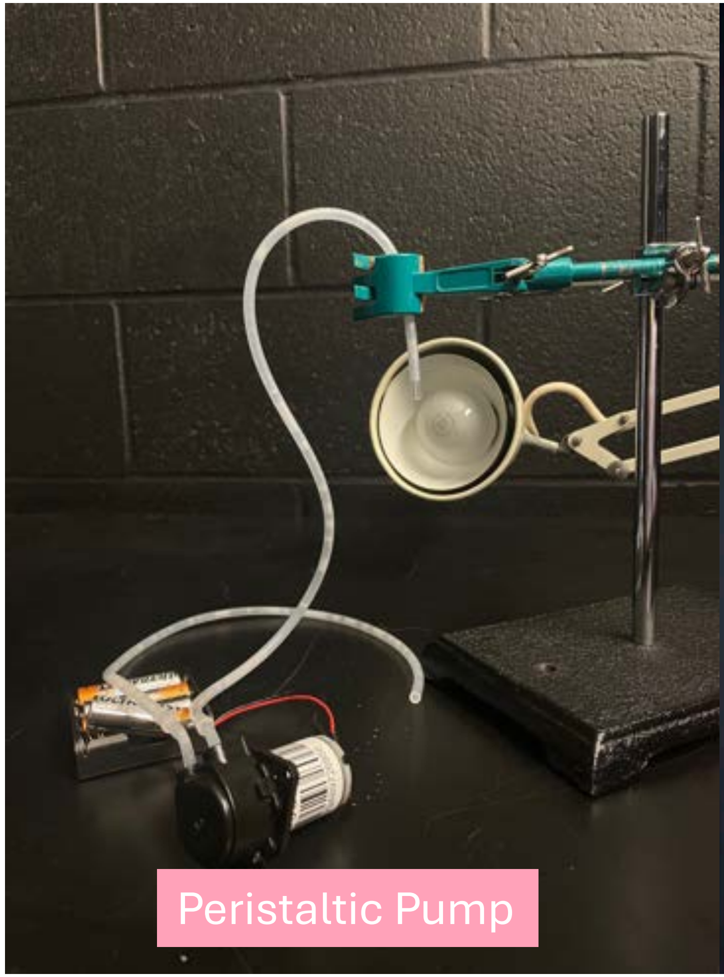
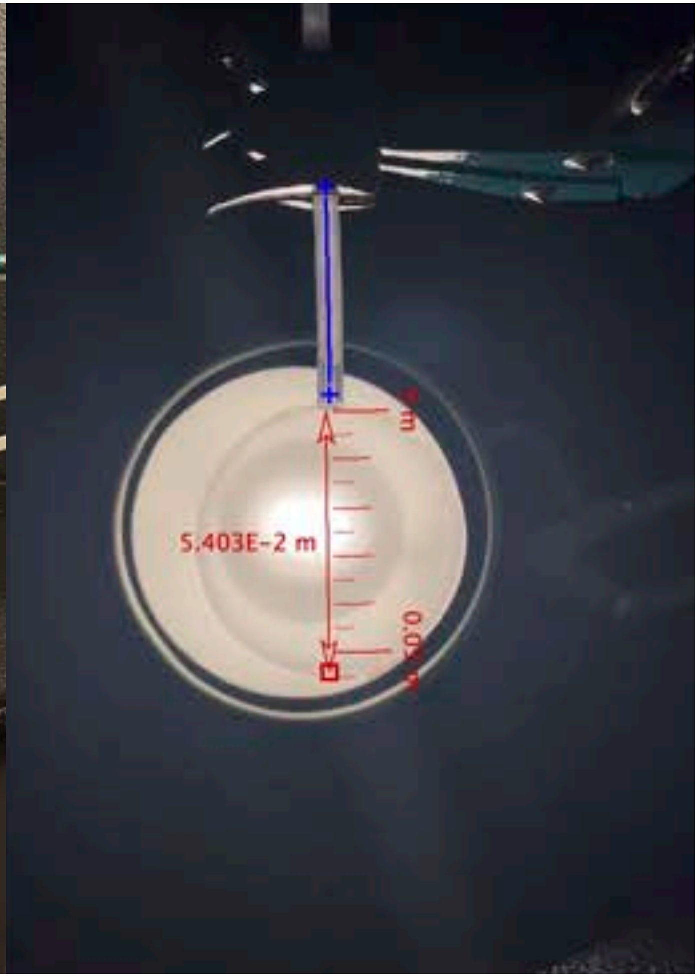


Photo from © ASTC Science World Society 2025

What is the critical surface area?





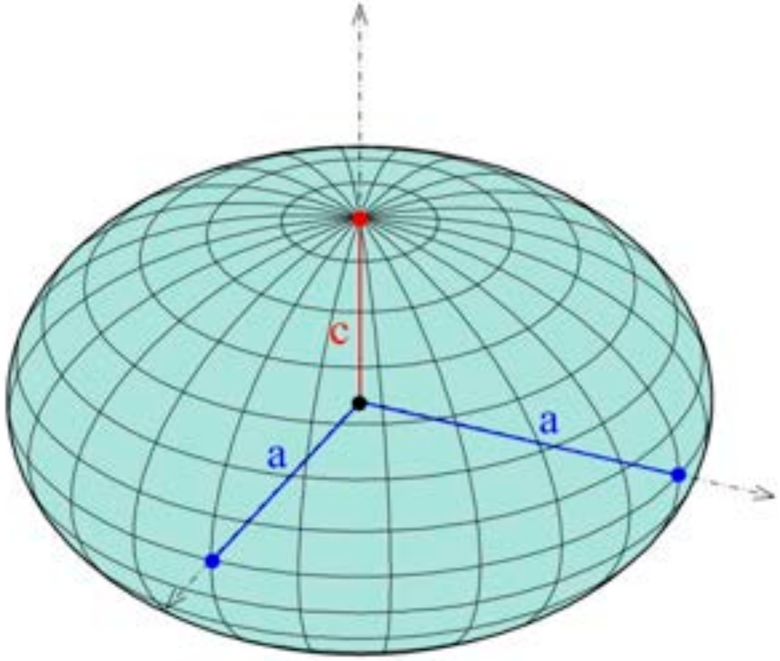
Maximum Radius

$R_{crit} = 2.72 \pm 0.18\ cm$

Maximum Surface Area

$S_{crit} = 93.0 \pm 12.3\ cm^2$

Surface Area of Oblate Spheroid



Oblate Spheroid

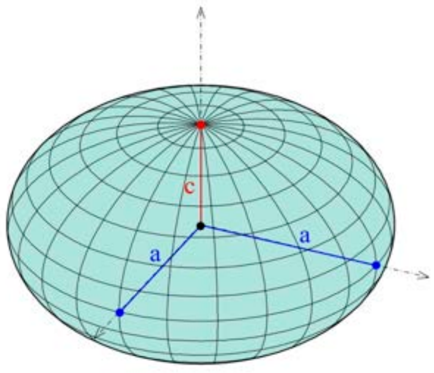
Credit: By Ag2gaeh (Own work) [CC BY-SA 4.0 (https://creativecommons.org/licenses/by-sa/4.0)], via Wikimedia Commons

$$S = 2\pi a^2 + \frac{\pi c^2}{e} \ln\left(\frac{1+e}{1-e}\right)$$

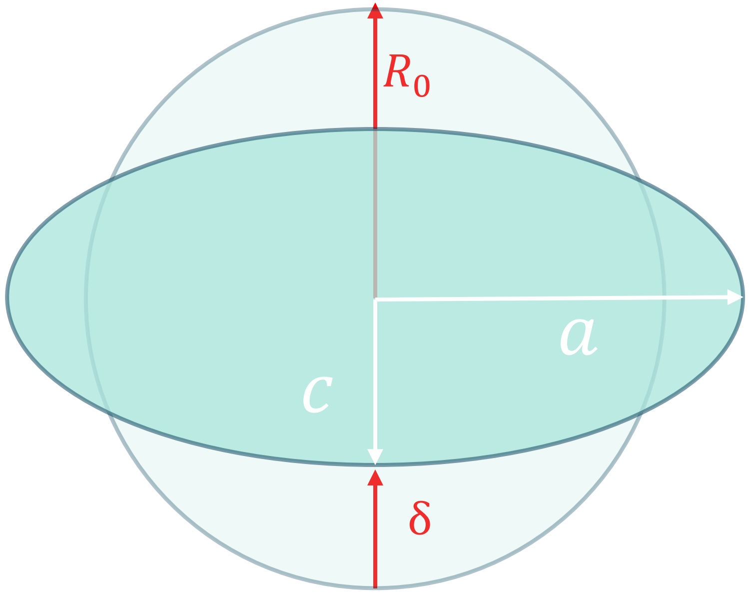
$$e = \sqrt{1 - c^2/a^2}$$

Beyer, W. H. *CRC Standard Mathematical Tables, 28th ed.* Boca Raton, FL: CRC Press, 1987.

Surface Area of Oblate Spheroid



Conservation of Volume:



$$c = R_0 - \delta$$

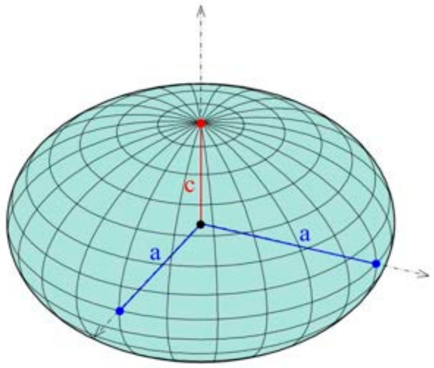
$$a = \frac{R_0^{3/2}}{\sqrt{R_0 - \delta}}$$

$$V_0 = 30\, ml$$

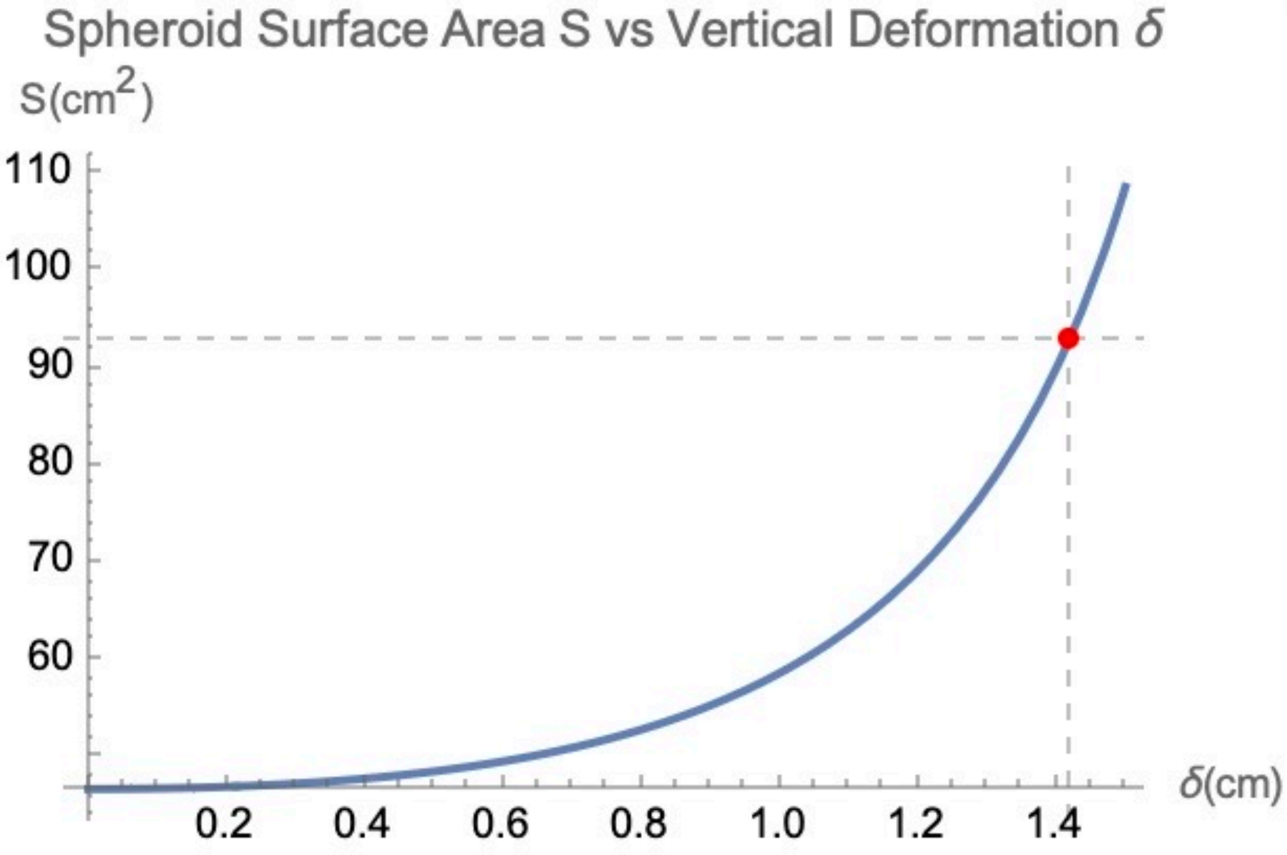
$$R_0 = 1.93\, cm$$

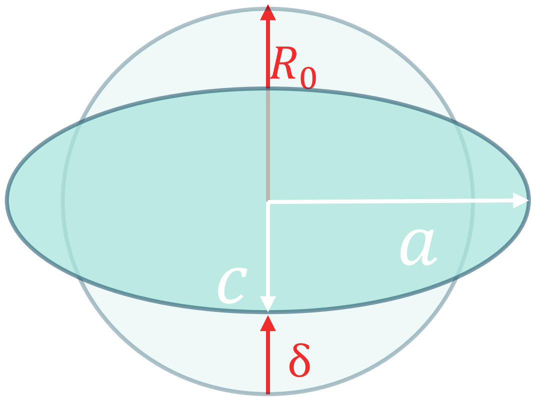
Beyer, W. H. *CRC Standard Mathematical Tables, 28th ed.* Boca Raton, FL: CRC Press, 1987.

Maximum Vertical Deformation



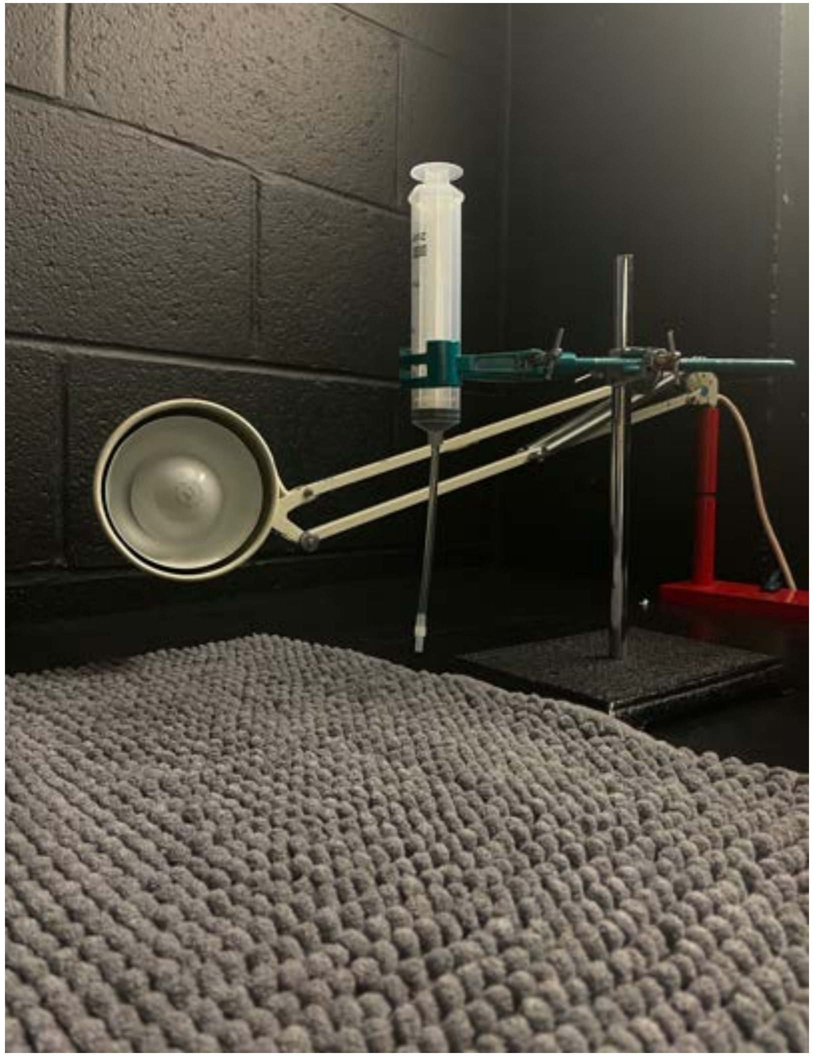
$$S = 2\pi a^2 + \frac{\pi c^2}{e}\ln\left(\frac{1+e}{1-e}\right) = S_{crit}$$





$$\delta_{cric} = 1.42 \pm 0.09\ cm$$

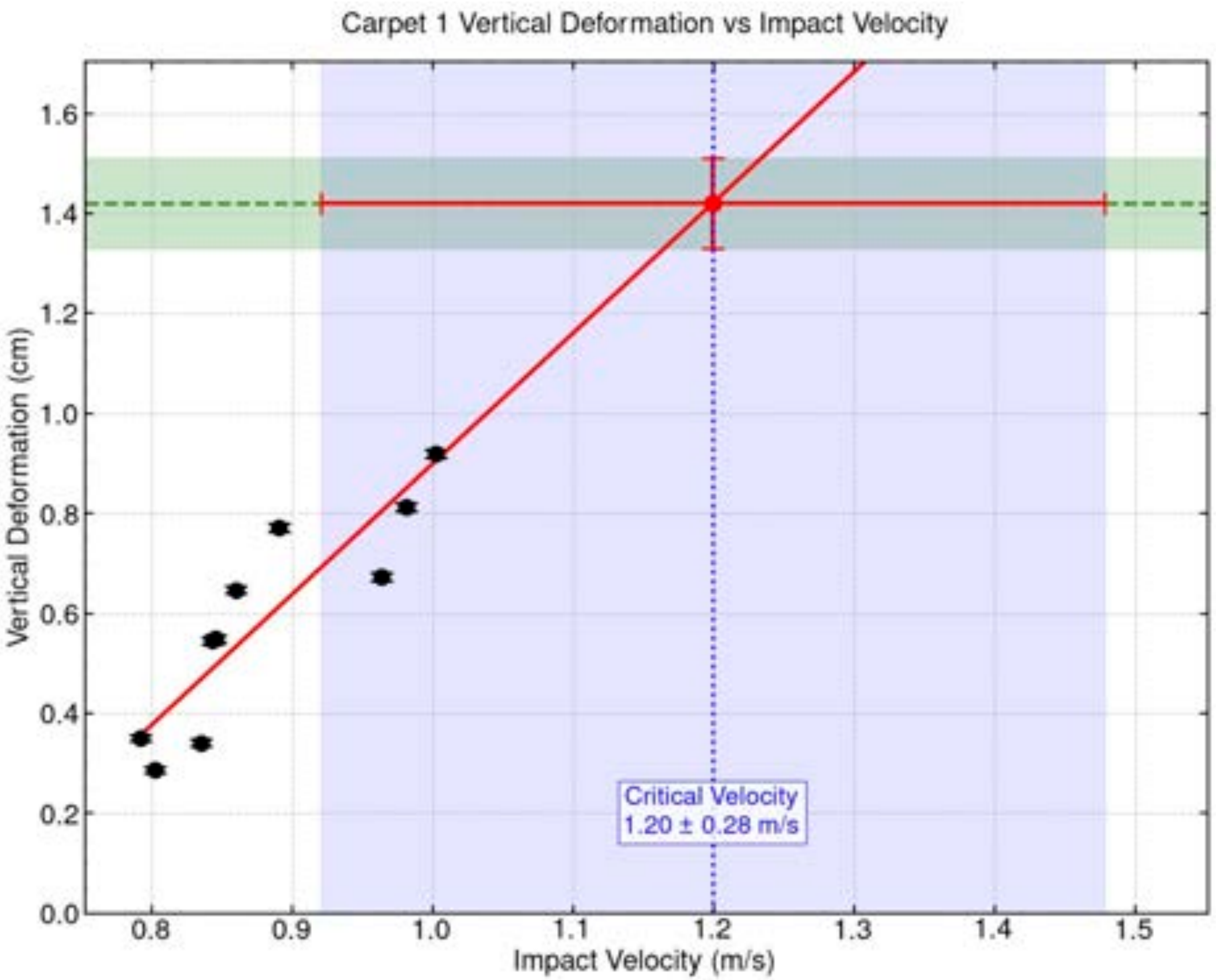
Maximum Vertical Deformation



Predict Critical Velocity

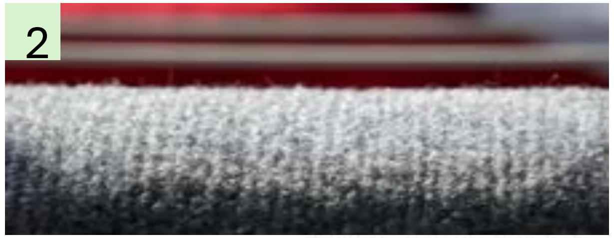
- Deformation of the soap bubble suggest a linear relation with landing velocity [1]

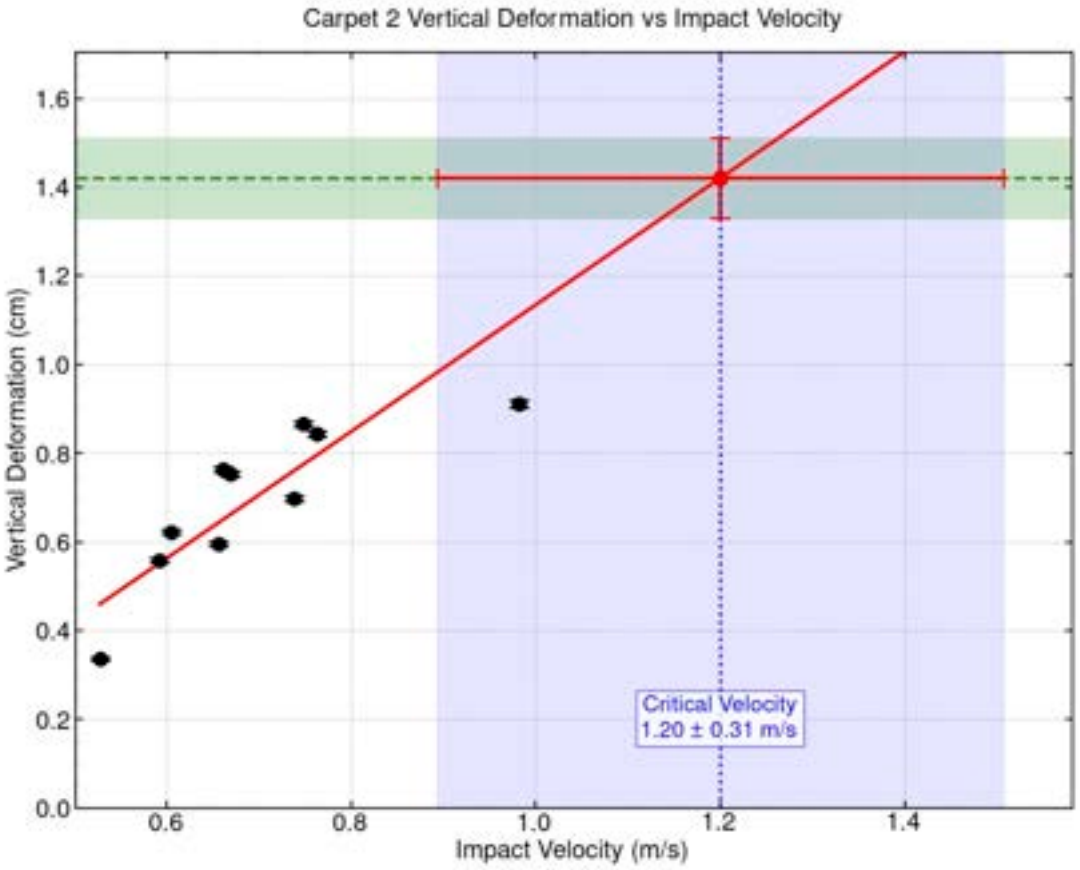
1



Carpet Samples

2





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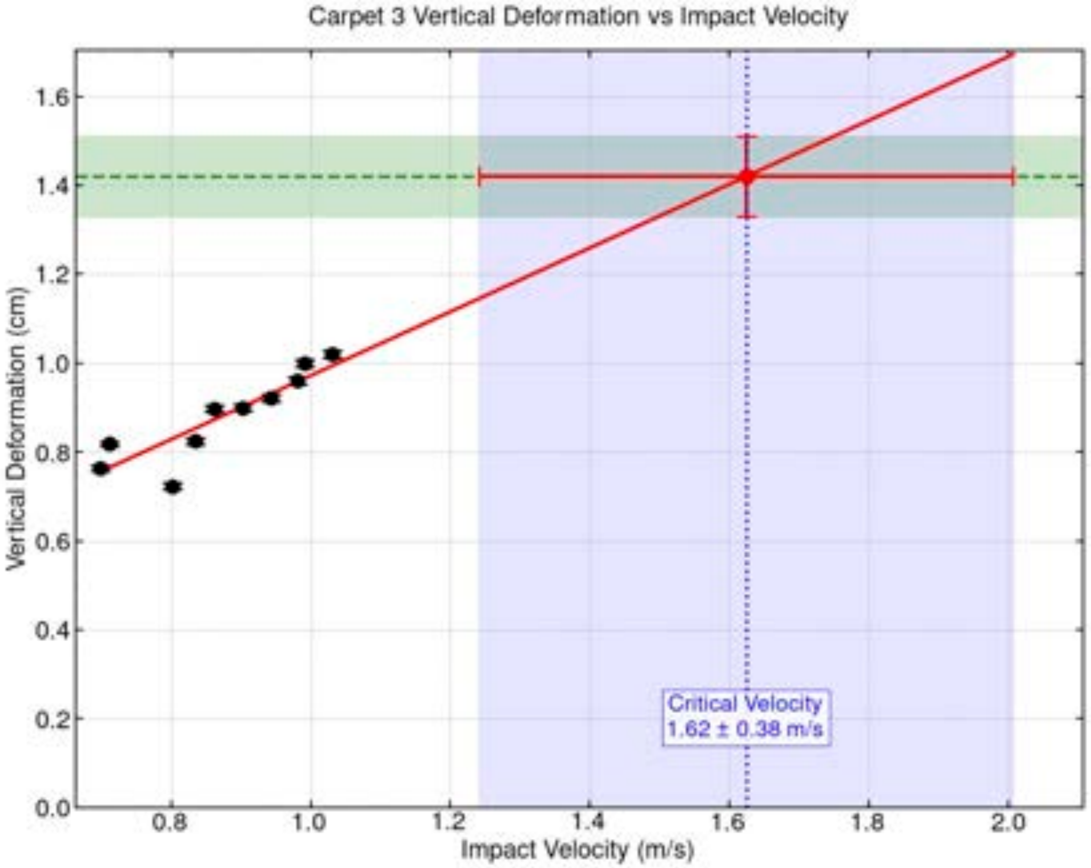

Conclusion

- Determine the **maximal surface area** of a soap bubble
- Find the $\boldsymbol{\delta_{cric}}$ of the oblate spheroid using the maximum surface area and **conservation of volume**
- Determine the **linear relation** of any given carpet between the vertical deformation and the landing velocity
- Find $\boldsymbol{V_{cric}}$ **using** $\boldsymbol{\delta_{cric}}$

References

- **de Gennes, P.-G., Brochard-Wyart, F., & Quéré, D.** (2004). *Capillarity and wetting phenomena*. Springer. https://doi.org/10.1007/978-0-387-21656-0
- **Beyer, W. H.** (1987). *CRC standard mathematical tables* (28th ed.). CRC Press.
- **Iqbal, S.** (2023). Fabrication and characterization of durable superhydrophobic and superoleophobic surfaces on stainless steel mesh substrates. *Preprints*. https://doi.org/10.20944/preprints202305.1234.v1 *(Note: Replace with actual DOI if available)*
- **Science World.** (n.d.). *Bubbles*. Retrieved from https://www.scienceworld.ca/resource/bubbles/
- **Vos, V. S. S.** (2012). *Deformations of a bubble* [Bachelor's thesis, Universiteit Leiden].

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- McGill Science Undergraduate Society
- McGill Crowdfunding

Other Resources:
- McGill Maker Space

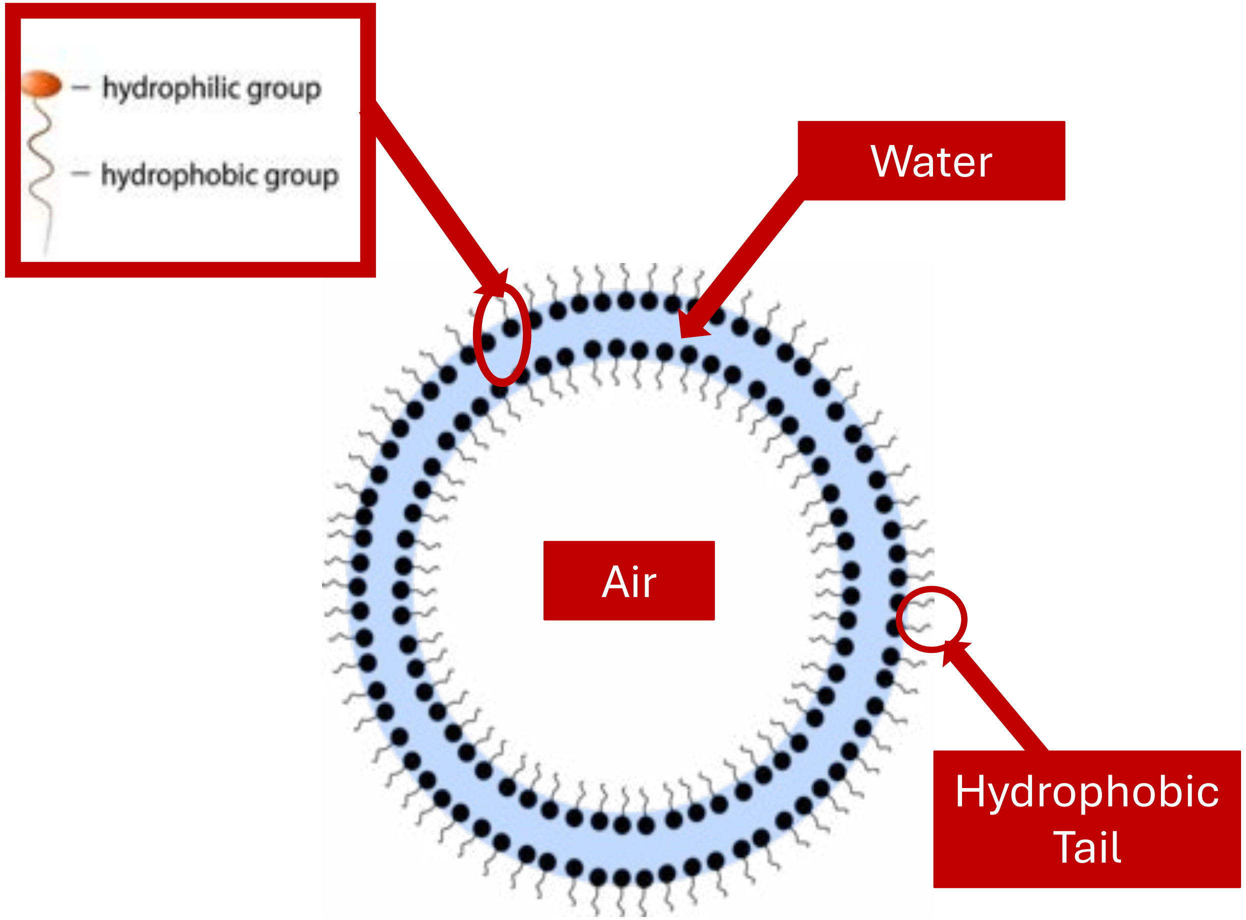
McGill International Physicists' Tournament

What makes a soap bubble?

A soap bubble consist of:

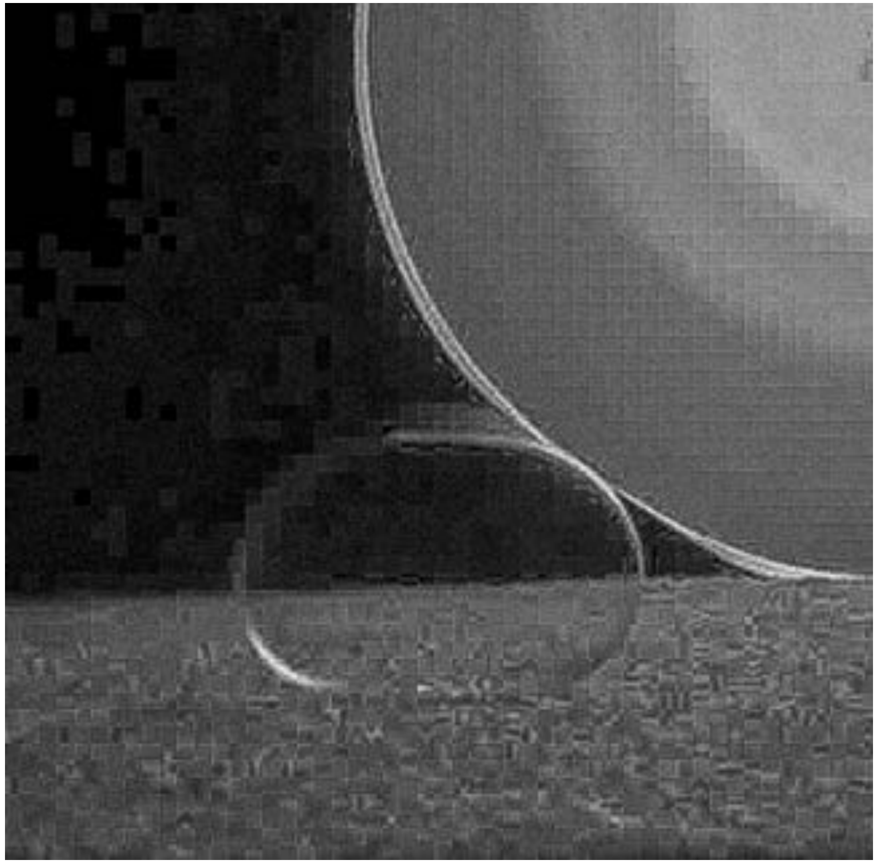
- A thin layer of water sandwiched between two layer of soap molecules
- Soap molecules have hydrophilic (water-loving) and hydrophobic (water-repelling) tails

This structure create surface tension which allows the bubble to hold its shape



Minimal Deformation at Low Velocity

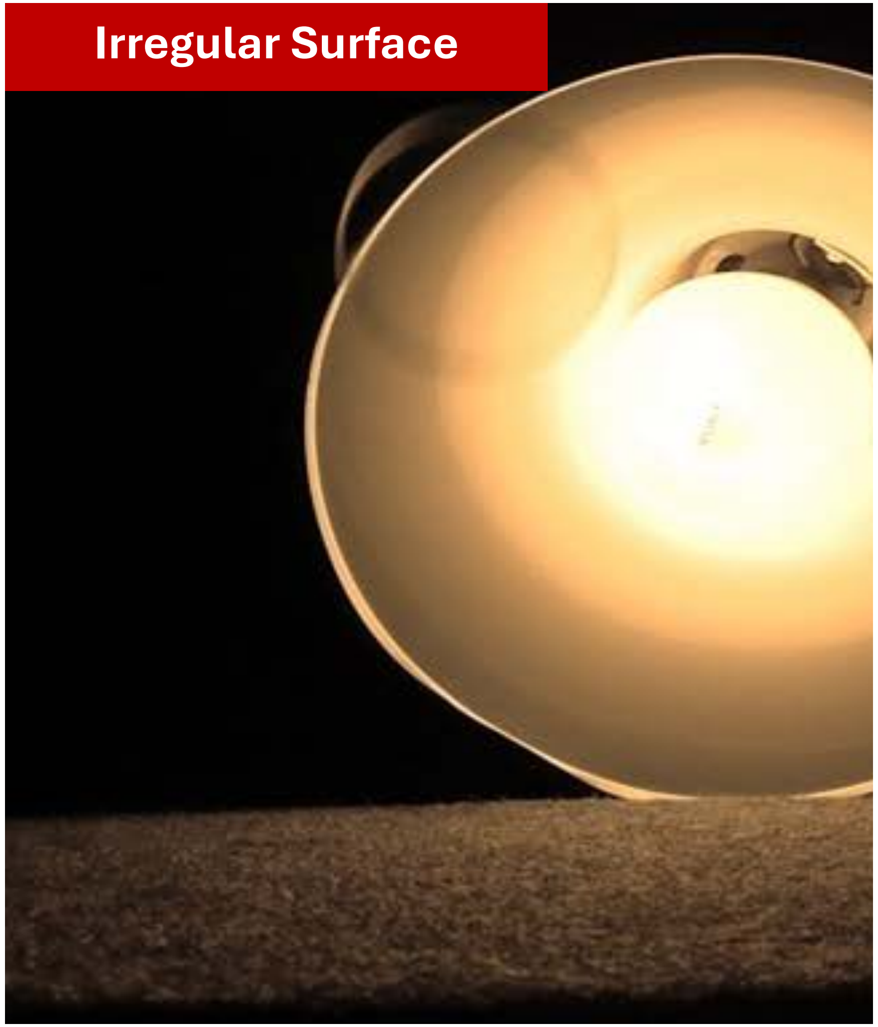
- When a soap bubble falls from a short distance (low velocity):
- Minimal deformation occurs
- The bubble vibrates slightly, with waves propagating from the bottom to the top
- Minimal change in internal pressure, preserving the bubble's spherical shape
- Why does it matter?
- A sphere minimizes surface energy, maintaining equilibrium and preventing rupture



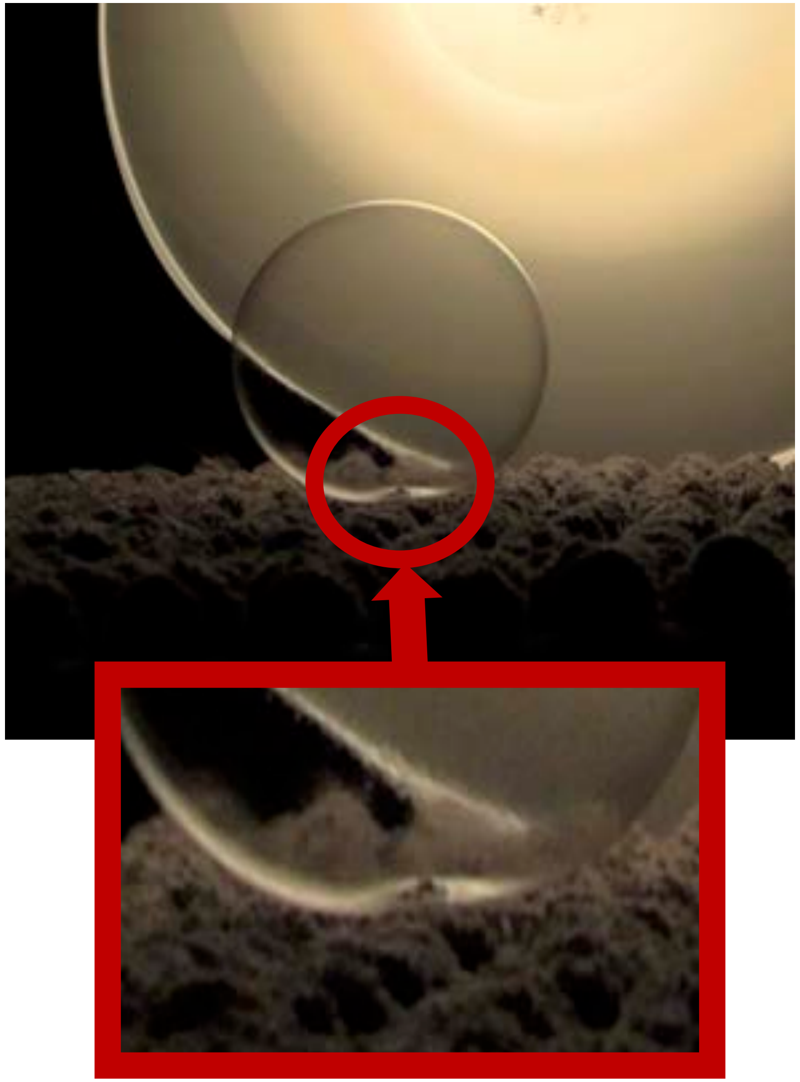
Surface Roughness

Smooth Surface

Irregular Surface



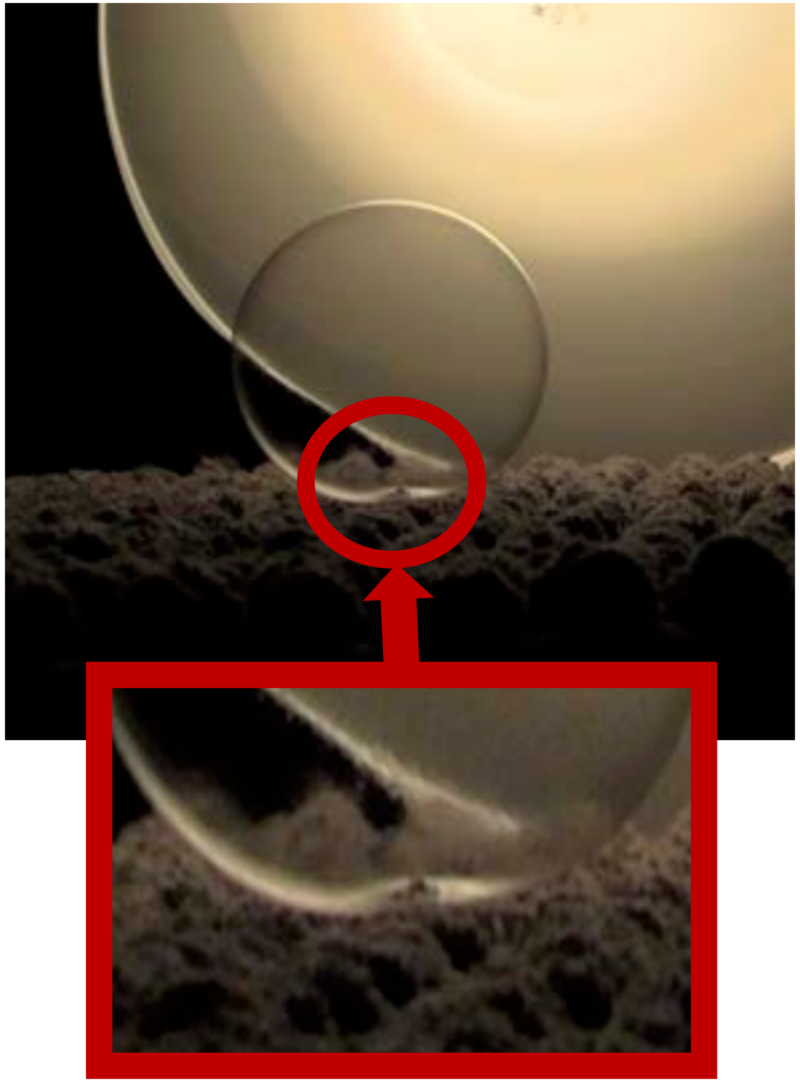
Air Pocket Upon landing



- Upon landing, the soap bubble compresses, increasing air pressure beneath it
- An air film forms between the bubble and the carpet
- Soft fiber permits the carpet to distribute the air and land smoothly

[1] Chen et al- *Droplet Impact on Soft Viscoelastic Surfaces*. Physical Review E94, 063117(2016)

Surface Meniscus



- Upon landing, the soap bubble compresses, increasing air pressure beneath it
- An air film forms between the bubble and the carpet
- This air film:
- 1. Reduces surface energy
- 2. Allow the bubble to rebound and stabilize
- Meniscus Formation: The curved liquid-air interface helps stabilize the bubble

[1] Chen et al- *Droplet Impact on Soft Viscoelastic Surfaces*. Physical Review E94, 063117(2016)

Young-Dupre Equation Derivation 1

Derivation of the Young-Dupré Equation

The Young-Dupré equation relates the **work of adhesion** (W_{SL}) to the **surface tension of the liquid** (γ_L) and the **contact angle** (θ_Y).

1. Young's Equation

At equilibrium, the balance of surface tensions at the solid-liquid-gas interface is given by Young's equation:

$$\gamma_S = \gamma_{SL} + \gamma_L \cos\theta_Y$$

where:

- γ_S = surface free energy of the solid,
- γ_{SL} = solid-liquid interfacial tension,
- γ_L = surface tension of the liquid,
- θ_Y = Young's contact angle.

Young-Dupre Equation Derivation 2

2. Dupré Equation (Work of Adhesion)

The work of adhesion (W_{SL}) is defined as the energy required to separate the solid-liquid interface per unit area:

$$W_{SL} = \gamma_S + \gamma_L - \gamma_{SL}$$

3. Substitute γ_S from Young's Equation into Dupré

From Young's equation, we express γ_S as:

$$\gamma_S = \gamma_{SL} + \gamma_L \cos\theta_Y$$

Substituting into the Dupré equation:

$$W_{SL} = (\gamma_{SL} + \gamma_L \cos\theta_Y) + \gamma_L - \gamma_{SL}$$

4. Simplification

The γ_{SL} terms cancel out:

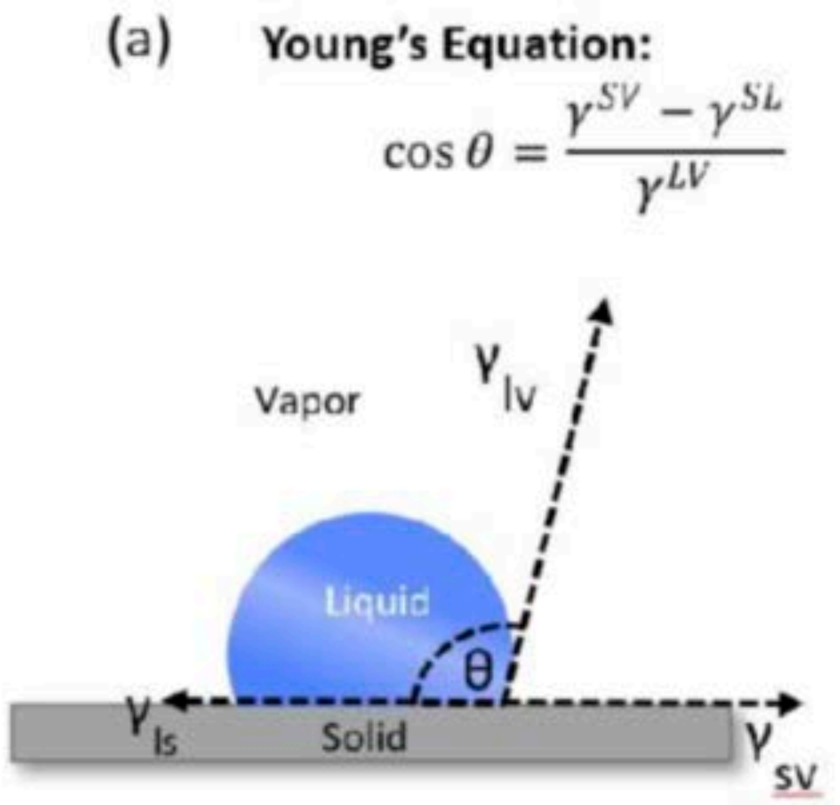
$$W_{SL} = \gamma_L \cos\theta_Y + \gamma_L$$

Factor out γ_L:

$$W_{SL} = \gamma_L(1 + \cos\theta_Y)$$

Capillary Adhesion Force

- Capillary adhesion force arises from surface tension and capillary action
- Formula: $F_{adhesion} = \gamma L cos\theta$
- γ = surface tension
- L = contact line / triple line length
- θ = contact angle
- Key insight: As θ increases, adhesion force decreases, reducing the pull on the bubble



1: **Young-Laplace equation** and **surface tension mechanics**

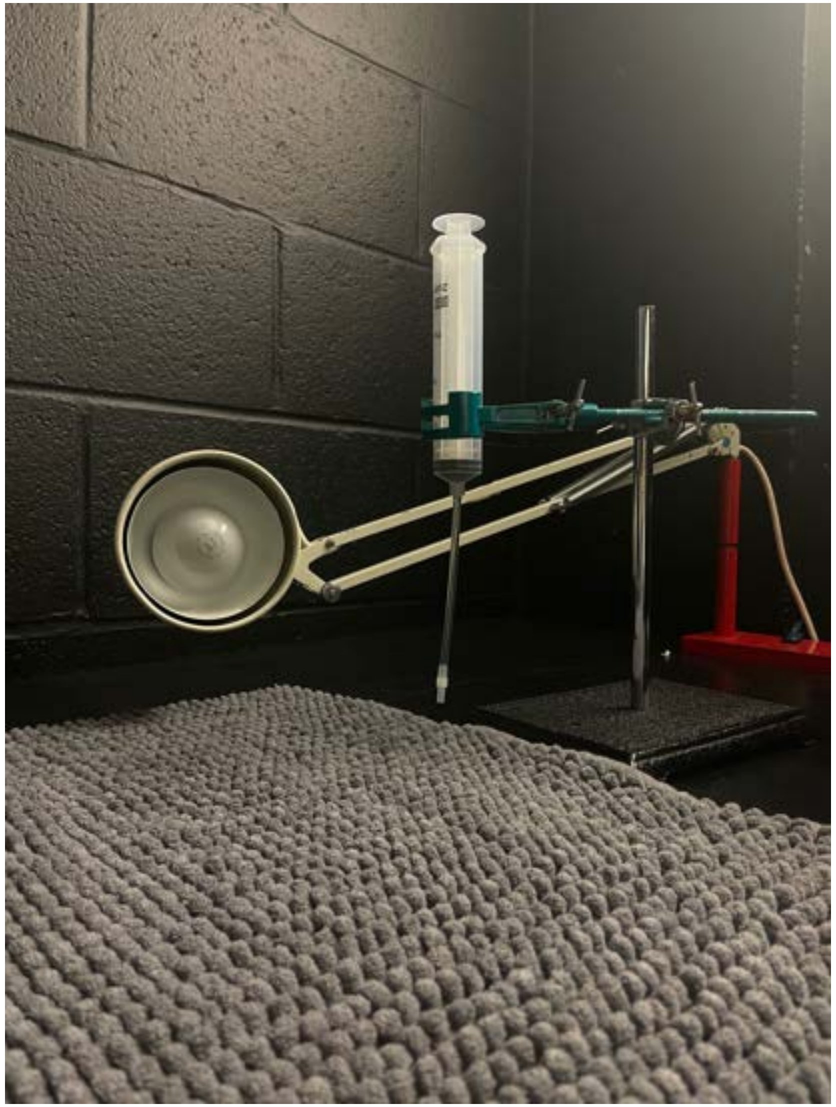
[a] Iqbal, Saira - *Fabrication and Characterization of Durable Superhydrophobic and Superoleophobic Surfaces on Stainless Steel Mesh Substrates*. 10.20944 (2023)

Critical Deformation (Mathematica)

```
In[1]:= (*Solve for the maximum deformation using surface area*)
(*Maximum Surface Area of the Sphere*)
ClearAll["Global`*"];
(*Define constants*) s = 4 * Pi * 2.72 ^ 2;
r0 = 1.928;

(*Define functions of x*)
a[x_] := r0 ^ (3 / 2) / (r0 - x) ^ (1 / 2);
b[x_] := r0 - x;
e[x_] := Sqrt[1 - b[x] ^ 2 / a[x] ^ 2];
f[x_] := 2 * Pi * a[x] ^ 2 + Pi * (b[x] ^ 2 / e[x]) * Log[(1 + e[x]) / (1 - e[x])];

(*Solve for x*)
solution = FindRoot[f[x] == s, {x, 0.5}];
xSol = x /. solution;

(*Print results*)
Print["Solution: x = ", xSol];
Print["f(x) - s = ", f[xSol] - s];

Solution: x = 1.41956
f(x) - s = 1.42109 × 10^-14
```

Other Carpet Samples

Experimental Setup



Why can't we reach the critical velocity experimentally?

- The Lubrication Approximation of Navier-Stokes equation describes fluid motion
- Lubrication approximation:
- As carpet fibers get smaller, viscous forces ($F_{viscous}$) increase
- These forces oppose bubble downwards motion preventing sudden impact

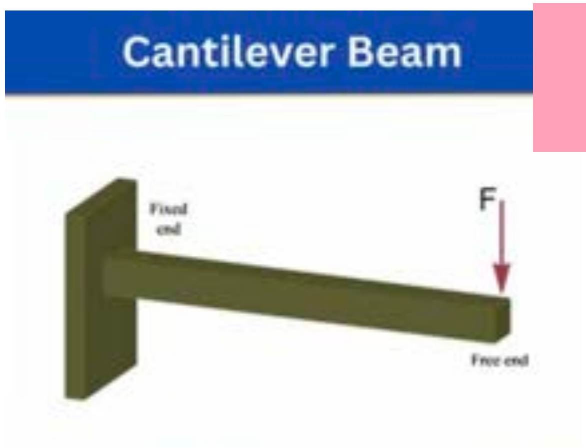
$$F_{viscous} = -\frac{3\pi\mu R^4}{2h_{air}^3}\frac{dz}{dt}$$

$h_{air} << r$

$\mu =$ air viscosity

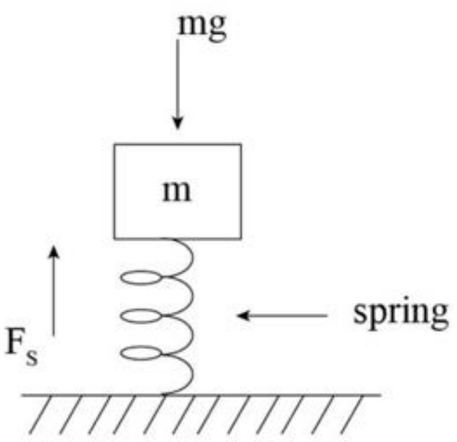
$h_{air} =$ air gap thickness

Suitable coefficient for Carpet Fiber



$$F_{\text{elastic}} = -kx$$

$$F_{\text{elastic}} \propto -n k_f \delta$$



Number of Fibers

ρ_f = fiber density

$n = \pi r^2 \rho_f$ = number of fiber in contact

Fiber Stiffness [1]

$$k_f = \frac{3EI}{L^3}$$

δ = fiber deflection

E = Young's modulus

I = moment of inertia

L = fiber length

1: Lateral stiffness of **Cantilever beam** (beam with one end supported)